

Ex 4.4. ANSWERS

1 a) $\cos \theta = \frac{\sqrt{3}}{2}$

$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta = 30^\circ \text{ (1st quadrant)}$$

$$\theta = 360 - 30 = 330^\circ \text{ (4th quadrant)}$$

$$\theta = 30^\circ, 330^\circ$$

b) $2 \sin \theta = \frac{1}{4}$

$$\sin \theta = \frac{1}{8}$$

$$\theta = \sin^{-1}\left(\frac{1}{8}\right) = 7.181^\circ \text{ (1st)}$$

$$\theta = 180 - 7.181 = 172.8^\circ \text{ (2nd)}$$

$$\theta = 7.181^\circ, 172.8^\circ$$

c) $\frac{1}{2} \tan \theta = -\frac{\sqrt{3}}{2}$

$$\tan \theta = -\sqrt{3}$$

$$\theta = \tan^{-1}(-\sqrt{3})$$

$$\theta = -60^\circ$$

$$\theta = 180 - 60$$

$$= 120^\circ \text{ (2nd)}$$

$$\theta = 360 - 60$$

$$= 300^\circ \text{ (4th)}$$

$$\theta = 120^\circ, 300^\circ$$

d) $2 \sin \theta - \sqrt{3} = 0$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta = 60^\circ \text{ (1st)}$$

$$\theta = 180 - 60$$

$$= 120^\circ \text{ (2nd)}$$

$$\theta = 60^\circ, 120^\circ$$

e) $4 \sin \theta + 2 = 0$

$$\sin \theta = -\frac{1}{2}$$

$$\theta = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = -30^\circ$$

$$\theta = 360 - 30 = 330^\circ \text{ (4th)}$$

$$\theta = 180 + 30 = 210^\circ \text{ (3rd)}$$

$$\theta = 210^\circ, 330^\circ$$

f) $2 \cos \theta = -\tan 45^\circ$

$$2 \cos \theta = -1$$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = 120^\circ \text{ (2nd)}$$

$$\theta = 180 + 60 = 240^\circ \text{ (3rd)}$$

$$\theta = 120^\circ, 240^\circ$$

2. a) $2 \cos^2 x - \cos x = 1$

$$\text{Let } \cos x = y$$

$$2y^2 - y = 1$$

$$2y^2 - y - 1 = 0$$

$$2y^2 - 2y + y - 1 = 0$$

$$2y(y-1) + (y-1) = 0$$

$$(2y+1)(y-1) = 0$$

$$y = 1 \text{ or } y = -\frac{1}{2}$$

$$\cos x = 1 \text{ or } \cos x = -\frac{1}{2}$$

$$x = \cos^{-1}(1)$$

$$= 0^\circ, 360^\circ$$

$$x = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$= 120^\circ$$

$$x = 180^\circ$$

$$= 240^\circ$$

$$x = 0^\circ, 120^\circ, 240^\circ, 360^\circ$$

$$b) 4 - 4\cos^2 x = 4\sin x - 1$$

$$4 - 4(1 - \sin^2 x) = 4\sin x - 1$$

$$4 - 4 + 4\sin^2 x = 4\sin x - 1$$

$$4\sin^2 x - 4\sin x + 1 = 0$$

$$\text{Let } \sin x = a$$

$$4a^2 - 4a + 1 = 0$$

$$4a^2 - 2a - 2a + 1 = 0$$

$$2a(2a-1) - (2a-1) = 0$$

$$(2a-1)(2a-1) = 0$$

$$2a-1=0 \text{ or } 2a-1=0$$

$$a = \frac{1}{2} \text{ or } a = \frac{1}{2}$$

$$\sin x = \frac{1}{2}$$

$$x = \sin^{-1}\left(\frac{1}{2}\right)$$

$$x = 30^\circ$$

$$x = 180 - 30^\circ$$

$$= 150^\circ$$

$$x = 30^\circ, 150^\circ$$

$$c) \sin^2 x - \cos^2 x = 0$$

$$(\sin^2 x) - (\cos^2 x) = 0$$

$$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = 0$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x - \cos^2 x = 0$$

$$\sin^2 x - (1 - \sin^2 x) = 0$$

$$\sin^2 x - 1 + \sin^2 x = 0$$

$$2\sin^2 x - 1 = 0$$

$$\text{Let } \sin x = t$$

$$2t^2 - 1 = 0$$

$$t^2 = \frac{1}{2}$$

$$t = \pm \sqrt{\frac{1}{2}}$$

$$t = \sqrt{\frac{1}{2}} \text{ or } t = -\sqrt{\frac{1}{2}}$$

$$\sin x = \sqrt{\frac{1}{2}} \text{ or } \sin x = -\sqrt{\frac{1}{2}}$$

$$x = \sin^{-1}\left(\sqrt{\frac{1}{2}}\right) \quad x = 45^\circ$$

$$x = 45^\circ, 135^\circ \quad x = 315^\circ$$

$$x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

$$\tan^2 x + \tan x - 2 = 0$$

$$\text{Let } \tan x = y$$

$$y^2 + y - 2 = 0$$

$$y^2 - y + 2y - 2 = 0$$

$$y(y-1) + 2(y-1) = 0$$

$$(y-1)(y+2) = 0$$

$$y = 1 \text{ or } y = -2$$

$$\tan x = 1 \quad \tan x = -2$$

$$x = \tan^{-1}(1) \quad x = \tan^{-1}(-2)$$

$$x = 45^\circ \quad x = -63.43^\circ$$

$$x = 180 + 45^\circ$$

$$= 225^\circ$$

$$x = 180 - 63.43^\circ$$

$$= 116.6^\circ$$

$$x = 360 - 63.43^\circ$$

$$= 296.6^\circ$$

$$x = 45^\circ, 116.6^\circ, 225^\circ, 296.6^\circ$$